SOLID STATE PHYSICS 1

- November 2001 -

Do not forget to write your full name and student number on each sheet.
Please use separate sheets for each of the three problems.

I. The zinc blende structure is a face-centered cubic Bravais lattice with cube side a. The conventional cell contains four positively charged ions at (0 , 0 , 0),
(½ , ½ , 0), (½ , 0 , ½), and (0, ½ , ½), and four negatively charged ions at (¼ , ¼ , ¼) , (¾ , ¾ , ¼), (¾ , ¼, ¾), and (¼, ¾ , ¾).

a. What is the general definition of a primitive cell
b. What is the volume of the primitive cell for the zinc blende structure
c. If we define the miller indices in units of the conventional unit cell, what is the distance between the (111) plane and the (222) plane.
d. What is the angle between an incoming X-ray beam and the first order Bragg reflection from the {111} planes (use a = 10 Å; λ = 1 Å).

e. Given the reciprocal lattice vector $\mathbf{k} = 4\pi/a\cdot(h\,\hat{\mathbf{x}}+k\,\hat{\mathbf{y}}+l\,\hat{\mathbf{z}})$ show that for integer h, k, and l the structure factor S_k is equal to $4(f_p + f_n)$ if $h + k + l$ is even and $4(f_p - f_n)$ if $h + k + l$ is odd.
Here, f_p and f_n are the atomic form factors for the positively and negatively charged ions, respectively.

II. Vanadium is a metal which crystallizes in a body-centered cubic structure (cube side a=3.02 Å). The electronic configuration of a free V atom is [Ar] $3d^3 4s^2$.

a. Derive an expression for the Fermi-energy in vanadium metal within the free electron approximation.
b. Derive an expression for the density of states at the Fermi-energy.
c. The presence of an external magnetic field B changes the density of states for spin up and spin down electrons. Make a sketch of the density of states in an external magnetic field.

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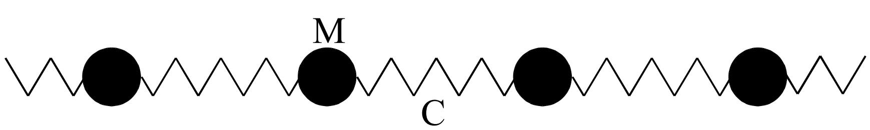
d. Derive the paramagnetic susceptibility of the conduction electrons in Vanadium (Pauli paramagnetism).
e. The electronic configuration of a free V^{3+} ion is [Ar]$3d^2$. Apply the Hund's rules to determine the ground state of this ion. Write the result in spectroscopic notation.
f. Describe the difference between paramagnetism, ferromagnetism and anti-ferromagnetism.

III. Consider a linear chain consisting of identical atoms with mass M, connected by identical springs with spring constant C (see figure). Assume that each atom interacts with its nearest-neighbour atom only, and that this interaction is linear in the relative displacement along the chain.



a. Give the equation of motion of the atoms as a function of their displacement along the chain.
b. Calculate the phonon dispersion relation, and make a sketch of this.
c. Give an expression for the sound velocity along the chain.
d. Describe the physical meaning of the Debye temperature. Give an expression for the Debye temperature in the linear chain.
e. Derive an expression for the total phonon energy at low temperatures within the Debye approximation, and show that the heat capacity at low temperatures is linear in the temperature.

note: $$\int_0^\infty \frac{x}{e^x - 1} \cdot dx = \frac{\pi^2}{6}.$$